6/9/24:

True / False: (End of Chapter 7)

1. If 0 is an eigenvalue of A, then $\det A = 0$ True

2. If $\vec{v}$ is an eigenvector of A, then $\vec{v}$ is also an eigenvector of A^3. True

3. If 1 is the only eigenvalue of A, then $A = I$, the identity matrix. False

4. All diagonalizable matrices are invertible False

5. If $\vec{v}$ and $\vec{w}$ are eigenvectors of a matrix A for two different eigenvalues, then $\vec{v}$ and $\vec{w}$ are linearly independent. $\boxed{\text{True}}$

$$\boxed{\vec{v} = c\vec{w}}$$

$$A\vec{v} = \lambda_1 \vec{v}$$
$$A\vec{w} = \lambda_2 \vec{w}$$

$$\underset{\lambda_1 \vec{v}}{\underset{\parallel}{A\vec{v}}} = A c\vec{w} = cA\vec{w} = c \cdot \lambda_2 \vec{w}$$
$$= \lambda_2 \vec{v}$$

$$\lambda_1 = \lambda_2$$

(2) If $\vec{v}$ is an e.v. of A, is $\vec{v}$ is an eigenvector of A^3? $\boxed{\text{True}}$

$$A\vec{v} = \lambda\vec{v}$$

$$A^2\vec{v} = A(A\vec{v}) = A(\lambda\vec{v}) = \lambda \cdot A\vec{v} = \lambda^2\vec{v}$$

$$\vdots$$

$$A^n\vec{v} = \lambda^n\vec{v}$$

(3) If 1 is the only eigenvalue of a matrix A, then $A = I$ $\boxed{\text{False}}$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

① If 0 is an eigenvalue, then $\det(A) = 0$

$$E_0(A) = \ker(A)$$

Then there is $\vec{v} \neq \vec{0}$ s.t. $A\vec{v} = 0$

$\Rightarrow \ker(A) \neq \vec{0}$

$\Rightarrow A$ <u>not</u> invertible

$\Rightarrow \det(A) = 0$

$$\boxed{\begin{gathered} 0 \text{ is an eigenvalue} \\ \Updownarrow \\ \det(A) = 0 \end{gathered}}$$

$$\boxed{\det(A) = 0 \iff A \text{ \underline{not} invertible}}$$

A invertible $\iff$ A has n pivots $\iff$ A has <u>no</u> free variables
$(n \times n)$
$\iff \ker(A) = \vec{0}$

$$\boxed{\text{Full rank} = \text{invertibility} \quad (\text{for a square matrix } A)}$$

(4) All diagonalizable matrices are invertible

False

Diagonalizable $\iff$ For every eigenvalue λ, a.m. of λ = g.m. of λ

Invertibility $\iff$ All eigenvalues are non-zero

Ex:

$$A = B \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{D} B^{-1}$$

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$ is diagonalizable but not invertible

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$ is invertible but not diagonalizable

(4)

$$\underset{A}{\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}} \qquad \underset{A - \lambda I}{\begin{bmatrix} 1-\lambda & 2 & 3 \\ 2 & 3-\lambda & 4 \\ 3 & 4 & 5-\lambda \end{bmatrix}}$$

$$\det \begin{bmatrix} 1-\lambda & 2 & 3 \\ 2 & 3-\lambda & 4 \\ 3 & 4 & 5-\lambda \end{bmatrix}$$

$$(1-\lambda) \cdot \det \begin{pmatrix} 3-\lambda & 4 \\ 4 & 5-\lambda \end{pmatrix} - 2 \cdot \det \begin{pmatrix} 2 & 3 \\ 4 & 5-\lambda \end{pmatrix}$$

$$+ 3 \cdot \det \begin{pmatrix} 2 & 3 \\ 3-\lambda & 4 \end{pmatrix}$$

$$(1-\lambda)\Big((3-\lambda)(5-\lambda) - 16 \Big) - 2\Big(2(5-\lambda) - 12 \Big)$$

$$+ 3\Big(8 - 3(3-\lambda) \Big)$$

$$(1-\lambda)(3-\lambda)(5-\lambda) - 16(1-\lambda) - 4(5-\lambda) + 24$$

$$+24 - 9(3-\lambda)$$

$$(3 - 4\lambda + \lambda^2)(5-\lambda) \cancel{-16} \cancel{+16\lambda} \cancel{-20} \cancel{+10\lambda} \cancel{+24}$$

$$\cancel{+24} \cancel{-27} \cancel{+9\lambda} \quad -$$

$$(\cancel{15} \cancel{-20\lambda} \cancel{+5\lambda^2} \cancel{-3\lambda} \cancel{+4\lambda^2} \cancel{-\lambda^3}) \cancel{-15} \cancel{+35\lambda}$$

$$\underline{-\lambda^3 + 9\lambda^2 + 12\lambda}$$

$$\left[\begin{array}{cc|cc} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{array}\right] \qquad \det\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} = \det A \cdot \det B$$

$$\begin{bmatrix} 1-\lambda & 1 & 0 & 0 \\ 1 & 1-\lambda & 0 & 0 \\ 0 & 0 & 1-\lambda & 2 \\ 0 & 0 & 2 & 1-\lambda \end{bmatrix} \qquad \det\begin{bmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix} \cdot \det\begin{bmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{bmatrix}$$

$$= \left((1-\lambda)^2 - 1\right)\left((1-\lambda)^2 - 4\right)$$

$$= \left(\lambda^2 - 2\lambda + 1 - 1\right)\left(\lambda^2 - 2\lambda + 1 - 4\right)$$

$$= \lambda(\lambda - 2) \cdot (\lambda^2 - 2\lambda - 3)$$

$$\lambda(\lambda-2)(\lambda-3)(\lambda+1)$$

(2) Gaussian elimination (row reduction)

$$\begin{array}{c} \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & -2 & 4 & -8 \end{bmatrix} \end{array}\begin{array}{l} \\ -(\mathrm{I}) \\ -(\mathrm{I}) \\ -(\mathrm{I}) \end{array}$$

$$\to \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 2 & 0 & 2 \\ 0 & 3 & 3 & 9 \\ 0 & -1 & 3 & -7 \end{bmatrix}\begin{array}{l} \\ /2 \\ \\ \\ \end{array} \to \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 3 & 3 & 9 \\ 0 & -1 & 3 & -7 \end{bmatrix}\begin{array}{l} \\ \\ -3(\mathrm{II}) \\ +(\mathrm{II}) \end{array}$$

$$\to \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & 3 & -6 \end{bmatrix}\begin{array}{l} \\ \\ \\ -(\mathrm{III}) \end{array} \to \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & -12 \end{bmatrix}$$

$$\det = 1 \cdot 1 \cdot 3 \cdot (-12)$$
$$= -36$$

$$\det = 2 \cdot (-36) = -72$$

⑥

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$\lambda = -2, 0, 2$

$E_{-2}(A),\ E_0(A),\ E_2(A)$

$$E_\lambda(A) = \mathrm{Ker}(A - \lambda I)$$

$\underline{\lambda = -2}$:

$$\begin{array}{cc} \begin{bmatrix} 3 & 1 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix} & \begin{array}{l} /3 \\ \\ /3 \\ \\ \end{array} \end{array} \rightarrow \begin{array}{cc} \begin{bmatrix} 1 & 1/3 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} & \begin{array}{l} \\ -(\mathrm{I}) \\ \\ -(\mathrm{III}) \end{array} \end{array}$$

$$\rightarrow \begin{array}{c} \begin{bmatrix} 1 & 1/3 & 0 & 0 \\ 0 & 8/3 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{cccc} & & & x_4 \text{ free} \end{array} \end{array}$$

$x_4 = t, \quad t \in \mathbb{R}$

$x_3 + x_4 = 0$

$\Rightarrow \quad x_3 = -t$

$\frac{8}{3} x_2 = 0 \Rightarrow x_2 = 0$

$x_1 + \frac{1}{3} x_2 = 0 \Rightarrow x_1 = 0$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -t \\ t \end{bmatrix} = t \underbrace{\begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}}$$

basis vector for $\ker(A+2I)$

$$\Rightarrow \quad E_{-2}(A) = \text{span}\left\{ \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} = -2 \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ -1+3 \\ -1-1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ -2 \end{bmatrix}$$

$E_0(A)$:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{array}{l} \\ -(\text{I}) \\ \\ -(\text{III}) \end{array}$$

$E_0(A) = \text{Ker}(A)$

If $A\vec{v} = 0 \cdot \vec{v} = 0$

$$\rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

$x_2 = t$

$x_1 + x_2 = 0 \Rightarrow x_1 = -t$

$x_3 + 3x_4 = 0 \Rightarrow x_3 = 0$

$-4x_4 = 0 \Rightarrow x_4 = 0$

x_2 free

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -t \\ t \\ 0 \\ 0 \end{bmatrix} = t \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$E_0(A) = \text{span}\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

$\underline{\lambda = 2}$:

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 1 & -3 \end{bmatrix} \begin{matrix} \\ +(\mathrm{I}) \\ \\ +(\mathrm{III}) \end{matrix} \longrightarrow \begin{array}{c} \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{matrix} \quad & x_2 & \quad & x_4 \\ \quad & \text{free} & \quad & \text{free} \end{matrix} \end{array}$$

$x_2 = t$ $\quad t, s \in \mathbb{R}$

$x_4 = s$

$-x_3 + 3x_4 = 0$

$\Rightarrow x_3 = 3x_4 = 3s$

$-x_1 + x_2 = 0 \Rightarrow x_1 = x_2 = t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} t \\ t \\ 3s \\ s \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$E_2(0) = \text{span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix} =$$

$$\underbrace{\begin{array}{c} \begin{bmatrix} 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 3 \\ 1 & 0 & 0 & 1 \end{bmatrix} \\ \begin{array}{cccc} -2 & 0 & 2 & \end{array} \end{array} \cdot \begin{bmatrix} \boxed{-2} & 0 & 0 & 0 \\ 0 & \boxed{0} & 0 & 0 \\ 0 & 0 & \boxed{2} & 0 \\ 0 & 0 & 0 & \boxed{2} \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 3 \\ 1 & 0 & 0 & 1 \end{bmatrix}^{-1}}$$

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$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

upper-triangular

eigenvalue 1 :

alg- multi: 4

$$\chi_A(\lambda) = (\lambda - 1)^4$$

$\operatorname{Ker}(A - 1 \cdot I)$

$$A - 1I = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_2 = 0$

$x_3 = 0$

$x_4 = 0$

$x_1 = t$

x_1 free

$$t \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

eigenvector of e.v. 1

not diagonalizable

$$E_1(A) = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

g.m $= 1$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

upper triangular matrix

eigenvalues : 1, 2, 3, 4

Note : If a $n \times n$ matrix A has n distinct eigenvalues, then A is diagonalizable!

$\lambda = 1$:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 3 \end{bmatrix} \begin{matrix} \\ -(\mathrm{I}) \\ \\ \\ \end{matrix} \rightarrow \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 3 \end{bmatrix} \begin{matrix} \\ \\ -(\mathrm{II}) \\ \\ \end{matrix}$$

$$\rightarrow \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 3 \end{bmatrix} \begin{matrix} \\ \\ \\ -(\mathrm{III}) \end{matrix} \rightarrow \begin{array}{c} x_1 \text{ free} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{matrix} x_2 = 0 \\ x_3 = 0 \\ x_4 = 0 \\ \\ \end{matrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} t \\ 0 \\ 0 \\ 0 \end{bmatrix} = t \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad E_1(A) = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

$\lambda = 2$:

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix} - (I)$$

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{aligned} &-x_1 + x_2 = 0 \Rightarrow x_1 = x_2 = t \\ &x_3 = 0 \\ &x_4 = 0 \end{aligned}$$

x_2 free

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} t \\ t \\ 0 \\ 0 \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$E_2(A) = \text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

$\underline{\lambda = 3:}$

x_3 free

$x_3 = t$

$$A - 3I$$

$$\begin{array}{c} \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ /3 \\ -(\text{III}) \end{array} \longrightarrow \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} -2x_1 + x_2 = 0 \\ -x_2 + 2x_3 = 0 \\ x_4 = 0 \\ \\ \end{array}$$

$$x_2 = 2x_3 = 2t$$

$$x_1 = \frac{1}{2} x_2 = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} t \\ 2t \\ t \\ 0 \end{bmatrix} \qquad E_3(A) = \text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$\underline{\lambda = 4:}$

$$\begin{bmatrix} -3 & 1 & 0 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$-3x_1 + x_2 = 0 \Rightarrow x_1 = \frac{1}{3}x_2 = t$

$-2x_2 + 2x_3 = 0 \Rightarrow x_2 = x_3 = 3t$

$x_3 = 3x_4 = 3t$

x_4 free

$x_4 = t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} t \\ 3t \\ 3t \\ t \end{bmatrix} = t\begin{bmatrix} 1 \\ 3 \\ 3 \\ 1 \end{bmatrix}$$

$$E_4(A) = \text{span}\left\{ \begin{bmatrix} 1 \\ 3 \\ 3 \\ 1 \end{bmatrix} \right\}$$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1}$$

Orthogonality:

A (square) matrix A is orthogonal if:

$$\left[\ \bullet\ A^{-1} = A^T \quad \left(A^T A = A A^T = I \right) \right]$$

- The columns of A are orthonormal:
  - orthogonal
  - all have unit length

$$V = \text{span}\{\underbrace{\vec{v}_1, \dots, \vec{v}_n}_{\text{orthonormal basis for } V}\}$$

$$A = \begin{bmatrix} \vec{v}_1 & \dots & \vec{v}_n \end{bmatrix}, \qquad \text{Proj}_V = A A^T$$

$$\left[\begin{matrix} \frac{a^2}{a^2+b^2} \\ \\ \\ \end{matrix} \right.$$

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(a) $t^3 + 3t^2 - 4$

$$= t^3 - t^2 + 4t^2 - 4$$

$$= t^2(t-1) + 4\underbrace{(t^2-1)}_{(t+1)(t-1)}$$

$$= (t-1)\left(t^2 + 4(t+1)\right)$$

$$= (t-1)(t^2+4t+4) = (t-1)(t+2)^2$$

1 : multiplicity 1

-2 : multiplicity 2

(b) $t^4 - 2t^2 + 1 = (t^2-1)^2 = (t+1)^2(t-1)^2$

-1 : multi. 2

1 : multi. 2

(c) $t^3 \underbrace{-6t^2}_{-3\cdot 2\cdot t^2} \underbrace{+12t}_{+3\cdot 4t} - 8 = (t-2)^3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \qquad \begin{matrix} 2-\lambda & 0 & 0 \\ 0 & -\lambda & 1 \\ 0 & -1 & -\lambda \end{matrix}\Bigg]$$

$$(2-\lambda)\underbrace{(\lambda^2+1)}_{i,\,-i}$$

3 $$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & x \\ 4 & 9 & x^2 \end{bmatrix}$$ invertible ?

$$1 \cdot \det\begin{pmatrix} 2 & 3 \\ 4 & 9 \end{pmatrix} - x \cdot \det\begin{pmatrix} 1 & 1 \\ 4 & 9 \end{pmatrix}$$

$$+ x^2 \cdot \det\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$$

$$\Rightarrow 1 \cdot (18-12) - x(9-4) + x^2(3-2)$$

$$x^2 - 5x + 6 = 0 \qquad x = 2, 3 \quad A \text{ is } \underline{\text{not}} \text{ invertible}$$

$$(x-2)(x-3)$$